# The Spreadsheet Metaphor For Word Problems

**0. INTRODUCTION**. We illustrate how the spreadsheet metaphor simplifies the solution of common word problems. Age, mixture, solution, distance-speed, and rate of work problems are solved by the spreadsheet method. There is a final section on ratio, proportion, and variation, both direct and inverse.

Many students consider “Word problems” a very difficult topic . Indeed, this seems to be true. One must *understand* the problem by identifying what quantities are involved and the relations between them. *Hidden* quantities like products and totals add to the difficulty.

In reality, most problems are easily solved by organizing the data in “Spreadsheet form” and then filling in the blanks. A problem, written in a long paragraph form, is intimidating and confusing. The best strategy is to eliminate the words by organizing the facts in a spreadsheet. Creating a partially filled spreadsheet provides a standard beginning and blank entries hint at the next step. Also, the spreadsheet helps identify *hidden* quantities

After solving, if you substitute the numerical values for expressions you obtain a concrete spreadsheet which is the **true solution** of the problem. One can easily check that all conditions are satisfied. The concrete spreadsheet is what you display in a presentation.

We adopt the following conventions for displaying the steps in the spreadsheet method. Regular text (1, 2, x) displays values given in the problem statement, Italic text (*1*,*2*, *x*) displays values computed in a row due to a column relationship, and bold text (**1, 2, x**) is used to display column totals. Percentage is written as its equivalent decimal to facilitate computations. We give detailed row and column captions, but abbreviations may be may used.

Our solutions begin with their filled out spreadsheet, followed by the explanation of how it was filled in. They conclude with an equation derived from a spreadsheet identity.

## 1. AGE PROBLEMS.

**Problem 1.1.** *A father is 3 times as old as his son. In 7 years he will be twice as old as his son. How old is the father now?*

***Solution 1.1.***

| | Father | Son |
|---|---|---|
| Now | *3x* | x |
| + 7 Years | **3x + 7** | **x + 7** |

| | Father | Son |
|---|---|---|
| Now | 21 | 7 |
| + 7 Years | 28 | 14 |

We create a captioned (Father, Son, Now, + 7 Years) spreadsheet. We enter the son's age now as x. Next, we enter the father's age now as 3x. Now we can enter the ages of father and son 7 years from now as (respectively) 3x + 7 and x + 7, yielding the equation

$3x+7 = 2(x+7)$ which has x=7 as the solution.

Therefore, **The father's age now** (3x) is 21 (**Always answer the question asked!)**.

The solution spreadsheet is displayed on the right. It is easy to check that all conditions are satisfied. To conserve space, we will not display solution spreadsheets in subsequent problems.

## 2. MIXTURE PROBLEMS.

**Problem 2.1.** *How many pounds of regular coffee costing $4.00 a pound must be added with 10 pounds of Premium coffee costing $7.00 a pound to create a blend costing $6.00 a pound?*

***Solution 2.1.***

| | Total Cost = | Pounds * | Cost per Pound |
|---|---|---|---|
| Regular | *4x* | x | 4 |
| Premium | *70* | 10 | 7 |
| Blend | Σ **4x+70** | Σ **x+10** | 6 |

We are given *cost per pound* and *number of pounds*. *Total cost* is a hidden variable. Note that *total cost = pounds * cost per pound*. Also, *total cost* and *pounds* for the blend are calculated by summing the values for regular and premium (We put a Σ in the spreadsheet as a reminder)

Now, we enter $4, $7, and $6 as *cost per pound*, 10 as premium *pounds,* and x as regular *pounds.* Next, we enter *4x* and *70* as regular and premium *total cost.* Now, we sum columns to get **4x+70** and **x+10** as blend *total cost* and *pounds.* From *total cost = pounds * cost per pound* for the blend, we get the equation

$4x+70 = 6(x+10)$ which has x=5 as the solution.

**Problem 2.2.** *I blend $6 of regular coffee costing $3 per pound with $30 of Premium coffee costing $5 per pound. What is the cost of the blend to me per pound?*

***Solution 2.2.*** This problem illustrates the use of B= C/A instead of C= A*B.

| | Total Cost = | Pounds * | Cost per Pound |
|---|---|---|---|
| Regular | 6 | *2* | 3 |
| Premium | 30 | *6* | 5 |
| Blend | Σ **36** | Σ **8** | x |

We enter \$6 and \$30 as Total cost and \$3 and \$5 as Cost per Pound. Enter x as Cost per Pound of the Blend. Since *Pounds = Total cost /Cost per pound*, compute *Pounds as 2* and *6* (respectively). Summing columns , we get **36** and **8** as Blend *Total cost* and *Pounds* . From *total cost = pounds * cost per pound* for the blend, we get

$$8x = 36 \qquad \text{which has } x=4.50 \text{ as the solution.}$$

The same spreadsheet shell suffices to solve the below problems.

**Problem 2.3.** *Mixing 3 pounds of regular coffee costing \$2.00 with 2 pounds of Premium coffee costing \$8.00 creates a blend costing how much a pound?*

**Problem 2.4.** *A company mixed 3 pounds of regular coffee costing \$5.00 a pound with 2 pounds of Premium coffee, getting a blend costing \$9.00 a pound. How much did the premium coffee cost per pound?*

## 3. SOLUTION PROBLEMS.

**Problem 3.1.** *How many liters of an 80% alcohol-water solution must be added to 20 liters of a 60% solution to obtain a 75% solution?*

***Solution 3.1.***

| | Alcohol Amount = | Total Amount * | Percentage % |
|---|---|---|---|
| Original solution | *12* | 20 | .60 |
| Added | *.80x* | x | .80 |
| Mixture | Σ **12+.80x** | Σ **x+20** | .75 |

We are given *Percentage alcohol. Alcohol Amount* is a hidden variable. Note that *Alcohol Amount = Total Amount * Percentage*. Also, *Alcohol Amount* and *Total Amount* for the Mixture are calculated by summing the values for Original solution and Added, signified by a Σ in the spreadsheet.

Now, we enter the *Percentages* (.60, .80, .75) , 20 as the Original solution *Total Amount*, and x as the Added *Total Amount* . Next, we enter .60 * 20 = *12* as Original solution *Alcohol Amount* and *.80x* as Added *Alcohol Amount* . Now, we sum columns to get **12+.80x** and **x+20** as Mixture *Alcohol Amount* and *Total Amount.* From *Alcohol Amount = Total Amount * Percentage* for the Mixture, we get the equation

$$12 + .80x = .75(x+20) \qquad \text{which has } x=60 \text{ as the solution.}$$

**Problem 3.2.** *How many liters of pure water must be added to 10 liters of an 80% alcohol-water solution to obtain a 50% solution?*

**Hint 3.2.** No **e**xtra spreadsheet column for *Amount water* is needed. The added solution is 0% alcohol.

**Problem 3.3.** *A jeweler needs 100 grams of alloy that is 56% gold. He has alloys that are 50% gold and 80% gold. How much of each should he mix to get his desired alloy?*

**Hint 3.3.** *Amount Gold* is a hidden variable. Also, you can set up the spreadsheet with x and y as the desired alloy amounts, noting the additional equation (readable from spreadsheet) x+y=100 or (preferably) call the amounts x and 100-x.

**Problem 3.4.** *A jeweler wants to use up his 40 grams of an alloy that is 70% gold. He receives an order for 50 grams of alloy that is 60% gold. What kind of alloy (% gold) should he mix with the original alloy to complete the order?*

## 4. TRAVEL PROBLEMS.

**Problem 4.1.** *I drive my son to Austin at 60 MPH and he drives us back to Dallas at at 80 MPH. What is the average speed for the round trip?*

***Solution 4.1.*** Many students say the average speed is 70 MPH. Really, the average speed is a little less than 70 MPH because more time is spent traveling at the lower speed.

| | Distance = | Rate * | Time |
|---|---|---|---|
| To Austin | 240 | 60 | *4* |
| Back to Dallas | 240 | 80 | *3* |
| Round trip | Σ **480** | x | Σ **7** |

The distance is unknown and should be called D. We can develop the spreadsheet in terms of D and would find D cancels out (see Solution 4.1B). It is easier to set D= 240.

So, we enter distances 240 and speeds (Rate) 60 and 80. We call round trip speed x. Now we compute the times (from) *Time = Distance/ Rate,* getting *4* and *6.* Next Summing columns, we get Round trip *Distance* and *Time* of **480** and **7.** So,

$$7x = 480 \qquad \text{which has } x=68.57 \text{ as the solution.}$$

***Solution 4.1B.***

| | Distance = | Rate * | Time |
|---|---|---|---|
| To Austin | D | 60 | *D/60* |
| Back to Dallas | D | 80 | *D/80* |
| Round trip | Σ **2D** | x | Σ **D/60 + D/80** |

Yielding 2D = (D/60 + D/80**)x.** Multiplying both sides by 240,

480D = (4D + 3D**)x.**

Dividing both sides by D yields the equation of the original Solution 4.1.

**Problem 4.2.** *A plane flies 240 miles from Dallas to a private airport near Houston. It takes 4 hours flying into a headwind. The return trip flying with the wind takes only 3 hours. How fast does the plane fly without a wind and how fast is the wind blowing?*

***Solution 4.2.*** We use two variables, p for the plane speed and w for the wind speed. So the speed of the trip to Houston is p-w and the speed of the trip back to Dallas is p+w.

| | Distance = | Rate * | Time |
|---|---|---|---|
| Trip against wind | 240 | p-w | 4 |
| Trip with wind | 240 | p+w | 3 |

The row entries give the equations $4(p-w)=240$ and $3(p+w)=240$ which has $p=70$ and $w=10$ as the solution.

**Problem 4.3.** *I traveled 120 miles to my friend. If I had gone 10 MPH faster, I would have arrived an hour sooner. How fast did I drive?*

**Hint 4.3.** Derive the equations $r*t=120$ and $(r+10)(t-1) =120$ with solution $t=4,r=30$ (the other solution $t=-3,r=-40$ is discarded).

## 5. RATE OF WORK PROBLEMS.

**Problem 5.1.** *If a job can be done by Bob in 4 days, by Sue in 6 days , and by Larry in 12 days, how long would it take all of them working together to do the job?*

***Solution 5.1.*** To eliminate fractions we assume the job consists of 12 units,

| | Work = | Rate * | Time |
|---|---|---|---|
| Bob | 12 | *3* | 4 |
| Sue | 12 | *2* | 6 |
| Larry | 12 | *1* | 12 |
| Together | 12 | Σ **6** | x |

We enter 12 **units** for work for everyone and the times each takes for the 12 unit job, x being entered for the Together time . Next we calculate Rate *(3, 2, 1)* for each person. We sum the Rate column to get the Together Rate. So, the Together row yields

$6x = 12$ which has $x=2$ as the solution.

**Problem 5.2.** *Bob and Sue working together can mow the lawn in 12 minutes. Sue can mow the lawn in 10 minutes. How long does it take Bob to mow the lawn?*

***Solution 5.2.***

| | Work = | Rate * | Time |
|---|---|---|---|
| Bob | 1 | *1/x* | x |
| Sue | 1 | *1/10* | 10 |
| Together | 1 | **1/x + 1/10** | 8 |

We enter 1 for work (1 lawn) for everyone and the times each takes for the lawn, x being entered for the Bob time . Next we calculate Rate *(1/x , 1/10)* for each person. We sum the Rate column to get the Together Rate **1/x + 1/10.** So, the Together row yields

$$8(1/x + 1/10) = 1$$ which has x=40 as the solution.

## 6. DIRECT AND INVERSE VARIATION.

Direct and Inverse variation describe a simple relationship between variables x and y. When x and y vary directly we say *x and y are in proportion*. *Direct Variation* says the **ratio** of x to y is some constant C ( x/y=C) and *Inverse Variation* says the **product** of x to y is some constant C ( xy=C). Once we recognize that quantities are in proportion, problems are easily solved by constructing a table.

| | Situation 1 | Situation 2 |
|---|---|---|
| x | $x_1$ | $x_2$ |
| y | $y_1$ | $y_2$ |

**Theorem 6.1.**

1. *If x and y vary directly , cross-multiply and set equal,* $x_1 / y_1 = x_2 / y_2$.
2. *If x and y vary inversely , multiply down and set equal,* $x_1 y_1 = x_2 y_2$.

The below problems illustrate how the spreadsheet approach can help decide whether variation is direct or inverse.

**Problem 6.2.** *If 6 men can do the job in 10 days, how many men are needed to complete the job in 15 days?*

***Solution 6.2.***

| | Situation 1 | Situation 2 |
|---|---|---|
| m | 6 | x |
| d | 10 | 15 |

My first thought was men and days were proportional, yielding the **incorrect** If 6 /10 = x/15. Really the quantities are vary **inversely** . So 6* 10 = x*15 with solution x= 4.

***Solution 6.2B.***

| | Work (1 job) = | Men * | Days |
|---|---|---|---|
| Situation 1 | 1 | 6 | 10 |

| | | | |
|---|---|---|---|
| Situation 2 | 1 | x | 15 |

The row entries give the equations 6 *10 = 1 job and 15 x = 1 job, yielding
6 *10 = 15 x with solution x= 4.

**Problem 6.3.** *If 5 men can make 35 cars in 14 days, how many men are needed to make 42 cars in 6 days?*

***Solution 6.3.***

The number of cars c varies directly with the number of men m and varies directly with the number of days d (joint variation). Generalizing Theorem 6.1**,**

$$\mathbf{m_1\ d_1 / c_1 = m_2\ d_2 / c_2} \qquad \text{yielding} \quad 5*14/35 = 6x/42 \qquad \text{(So } x=14\text{).}$$

***Solution 6.3B.*** *Rate=cars per man per day* is a hidden variable.

| | Cars = | Men * | Days * | Rate |
|---|---|---|---|---|
| Situation 1 | 35 | 5 | 14 | R |
| Situation 2 | 42 | x | 6 | R |

The row entries give the equations 35=5 *14*R and 42=6x *R , yielding x= 14 (divide sides of the second equation by the corresponding sides of the first equation).

**Problem 6.4.** *If 10 men can make 35 cars in 14 days, how many men who work twice as fast are needed to make 33 cars in 6 days?*