Four fold way

General Problem

Given a *Pool* of **n** objects. How many ways can they be used to fill **r** *blanks* (baskets, cages, etc).

2 questions :
1. Can objects be used more than once (repeats, ball back in urn) ? **Repeats**
2. Are the blanks considered different (ordered, named, colored) ? **Named**

Blanks / Pool	Named	No names
Repeats	n^r	${}_{n+r-1}C_r = {}_{n+r-1}C_{n-1}$
No Repeats	${}_nP_r$	${}_nC_r$

Questions:

1. How many committees of 3 people from a group of 9 people. **Ans.** ${}_9C_3$
2. How many committees of 3 people *with a president, vice president,, and secretary* from a group of 9 people. **Ans.** ${}_9P_3$
3. How many committees of 7 people *with a president, vice president,, and secretary* from a group of 9 people.

   Ans. ${}_9P_3 \bullet {}_{9-3}C_{7-3} = {}_9P_3 \bullet {}_6C_4 = {}_9C_7 \bullet {}_7P_3$

4. Also. **An Important Result:**

   If there are n subjects of which p_1 are alike of one kind; p_2 are alike of another kind; p_3 are alike of third kind and so on and p_r are alike of r^{th} kind, such that $(p_1 + p_2 + \ldots p_r) = n$.

   Then, number of permutations of these n objects is = $\dfrac{n!}{(p_1!).(p_2)!.....(p_r!)}$

Example: How many 11 letter words can be formed from Mississippi.

Note: 1 m, 4 I, 4 s, 2 p **Ans.** $\dfrac{11!}{1!4!4!2!}$

Note: ${}_nC_r = {}_nP_r\,/\,r!$ **Also,** ${}_nP_n = n!$ **Also,** ${}_nC_r = {}_nC_{n-r}$

Here and (4) above, divide because the order of the blanks (or identical elements) doesn't matter.

Bagel problems (pool allows repeats, blanks are no names) ***The above point of view doesn't work. We change to a new pool of choices (gaps) and the blanks are vertical line choices needed.*** **Explained below.**

An easy Bagel problem: How many ways can you choose 5 bagels if there are 3 types if each type is chosen **at least** once. ? Note we are looking for 3 positive (>0) integers that add up to 5, $x_1+x_2+x_3=5$. **Ans.** ${}_{5-1}C_{3-1}$

Explain: Here are the 5 bagels * * * * * . Note there are 4 (5-1) gaps. To get 3 numbers , we need 2 (3-1) vertical lines | in the gaps . Take the number of bagels to the left of each line (3-1) and the bagels to the right of the last line (+1), getting the 3 x. How many ways can you choose 2 gaps from a pool of 4 possibles ? **Ans.** ${}_4C_2$

In general: How many ways can you choose r bagels if there are n types if each type is chosen **at least** once. ? **Ans.** ${}_{r-1}C_{n-1}$

The Bagel problem: How many ways can you choose r bagels if there are n types (Some types may not be chosen) ? **Ans.** ${}_{n+r-1}C_r = {}_{n+r-1}C_{n-1}$

Explain: We add a special bagel for each type (n bagels added) and add n bagel choices to allow room for them.. Require choosing each special bagels. So there are n+r choices from n types. By the easy problem , there are $n+r-1$ gaps with $n-1$ vertical lines . So ${}_{n+r-1}C_{n-1}$ total possibilities.

The choices for the easy problem yield choices by taking away all the special bagels ■